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| **Question Pack – Math Geometry** |
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| **9/23/2007** |
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# Transformations

## Reflection about x,y axis

Px(x,-y)

P(x,y)

Py(-x,y)

Given the point P with coordinates (x,y). When reflected about the x axis, you will (x,-y) as shown in the figure. When reflected about the Y axis you get Py, which is (-x, y).

P270(-x,-y)

## Simple rotation about origin (900, 1800, 2700)

P90(y, -x)

P(x,y)

P180(-x,-y)

Given the point P with coordinates (x,y). When reflected about the x axis, you will (x,-y) as shown in the figure. When reflected about the Y axis you get Py, which is (-x, y).

## Translation

Tx

P(x,y)

P1(x+Tx,y+Ty)

Translation is moving the object in x,y. So if P is translated by (Tx, Ty). The final point P1 = (x+Tx, y+Ty).

Ty

# Line 2D

## Slope and distance between P1 & P2

Slope of a line (*m*) is defined as the ratio of change in y to the corresponding change in x.

P1(x1, y1)

P2(x2, y2)

(x2-x1)

(y2 - y1)

$$m=\frac{y\_{2}-y\_{1}}{x\_{2 }- x\_{1}}$$

All parallel lines have the same slope.

Perpendicular lines have the slope of $\frac{-1}{m}$

By applying Pythagoras theorem, we get the distance between P1 and P2.

$$P\_{1}P\_{2}=\sqrt{(x\_{2 }- x\_{1})^{2}+(y\_{2 }- y\_{1})^{2}}$$

## Equation of the line (slope/intercept form) $y=mx+b$

Y

X

b

y = mx + b

Where m is the slope of the line.

b is the y intercept as shown in the figure.

Slope of a line parallel to y axis is infinite.

Slope of a line parallel to x axis is zero.

To find the equation given 2 points, first find the slope. Then substitute the slope and one of the point in the above equation (*y = mx + b*) and find *b.*

## Dividing line segment

R

S

Y

X

k2

P1(x1,y1)

P(x,y)

P2(x2,y2)

k1

(y2-y1)

Point P divides P1 and P2 such P1P:PP2 = k1:k2 Because all side of ∆P1RP are parallel to corresponding sides in ∆PSP2 they are similar triangles. Hence

$$\frac{k\_{1}}{k\_{2}}=\frac{x-x\_{1}}{x\_{2}-x}$$

$$Hence, x=\frac{k\_{1}x\_{2}-k\_{2}x\_{1}}{k\_{1}+k\_{2}}$$

$$Simillarly, y=\frac{k\_{1}y\_{2}-k\_{2}y\_{1}}{k\_{1}+k\_{2}}$$

# Shapes 2D

## Triangle

### Types

#### Equilateral triangle

a

a

a

600

600

600

Equilateral triangle is the one that has all sides equal. Likewise all the angles of the triangle is same and equals to 600.

#### Isosceles triangle

b

a

a

θ

θ

α

Isosceles triangle is the one in which two sides and two angles are the same.

#### Scalene triangle

Scalene triangle has all three sides different.

### Area

If we know the base (b) and height (h) of the triange then

$$Area=\frac{bh}{2}$$

If the lengths of the trianges are a, b and c. Then

$$Area=\sqrt{s \left(s-a\right)\left(s-b\right)\left(s-c\right)}$$

Where s is semiperimeter, s = (a + b + c)/2

### Triangle Area give coordinates

Area of ∆ABC = Area of ABB1A1 - Area of BB1C1C + Area of CAA1C1

B1

A1

C1

B(x2,y2)

A (x1,y1)

C(x3,y3)

X

Y

$$=\left(x\_{1}-x\_{2}\right)\frac{\left(y\_{1}+y\_{2}\right)}{2}-\left(x\_{3}-x\_{2}\right)\frac{\left(y\_{2}+y\_{3}\right)}{2}+\left(x\_{3}-x\_{1}\right)\frac{\left(y\_{3}+y\_{1}\right)}{2}$$

$$=\frac{1}{2}\left[x\_{1}\left(y\_{2}-y\_{3}\right)+ x\_{2}\left(y\_{3}-y\_{1}\right)+ x\_{3}\left(y\_{1}-y\_{2}\right)\right]$$

### Collinear POINTS

Easiest way to find out if the 3 points are collinear is, if the area formed by the triangle is Zero. Hence

$$\left[x\_{1}\left(y\_{2}-y\_{3}\right)+ x\_{2}\left(y\_{3}-y\_{1}\right)+ x\_{3}\left(y\_{1}-y\_{2}\right)\right]=0$$

When the area is zero, means all three points are in a straight line.

### Triangle Centers

#### Incenter/INCircle

C

B

A

Circum Circle

InCircle

Intersection of the angular bisector is called the **Incenter**. Incenter is equidistant from 3 sides of the triange. A circle drawn from Incenter that touches the three sides of the triangle is called the **Incircle** or **Inscribed** circle. Incircle is the largest circle that can be drawn inside the triangle.

#### Circumcenter/Circumcircle

The point that is equidistant from the 3 vertices is called the **circumcenter**. This is the same as the intersection of perpendicular bisectors of the sides. Circle drawn from circumcenter such that it passes through 3 vertices is called circumcircle. This is the smallest circle that contains the triange.

#### Altitudes/Ortho Center

B

A1

P

C

A

The altitudes of a triangle are concurrent and intersect at **orthocenter**. Altitude is the line joining a vertex to the opposite side and is perpendicular to the opposite side. In the pictures AP is the altitude, AA1 is the median.

#### Median/Centroid

A1

C

B

A

C1

B1

G

c

a

c

a

b

b

**Median** is the line joining the vertex to the mid point of the opposite side.

In the triangle ABC, AA1 is the median, (i.e.) A1 is the midpoint for BC. Because the triangles AA1C and ABA1 share the same height (AP) and also their bases are same (BA1 and A1C) their areas are same as well. Thus the median divides the triangle into 2 triangles with equal areas.

G is the point where all the medians intersect. By similar reasoning as above we know that the areas for GBA1 and GA1C are the same, lets represent it as ‘a’. ABA1 and AA1C have the same area. So a + 2c = a + 2b. This means c=b. By similar reasoning it can be shown that a=c and b= c. Hence the medians divide the triangle into six equal triangles.

B1 is the mid point o AC. So B1=$(\frac{x\_{1}+x\_{3}}{2},\frac{y\_{1}+y\_{3}}{2}) $

B(x2,y2)

A (x1,y1)

A1

C(x3,y3)

C1

B1

G

Lets assume BG=2 x GB1. Basically G divides BB1 in the ratio of 2:1.

$$G=\left(\frac{2 x\frac{x\_{1}+x\_{3}}{2}+ x\_{2}}{2+1},\frac{2 x\frac{y\_{1}+y\_{3}}{2}+ y\_{2}}{2+1} \right)$$

$$G=\left(\frac{x\_{1}+x\_{2}+ x\_{3}}{3},\frac{y\_{1}+y\_{2}+ y\_{3}}{3} \right)$$

Same result is obtained when G is computed using AA1 or CC1. Hence we know that the medians intersect at 2/3 distance from the vertex. This intersection point is called **Centroid**.

## Quadrilateral

### Parallelogram (2 sets of parallel sides)

B1

D1

D

B

A

h

w

C

∆ AD1D and ∆ BCB1 are congruent. Because AD is parallel to BC and both share the same height. Hence ∆ BCB1 can be fitted as shown in the figure to form a rectangle. Thus Area

$$A=w×h$$

Diagonals of the parallelogram bisect.

Rhombus is a special parallelogram where all sides are same.

### Trapezium (Two sides are parallel)

h1

D

C

B

A

h2

w

$$Area of Trapezium ABCD= ∆ ABD+ ∆ BCD$$

$$= \frac{1}{2}h\_{2}×w+\frac{1}{2}h\_{1}×w $$

$$= w×\frac{h\_{1}+ h\_{2}}{2}$$

## Hexagon

A Hexagon has 6 equilateral triangles. If the side of the hexagon is a, then

$$Area=6\sqrt{(3a/2) \left(3a/2-a\right)\left(3a/2-a\right)\left(3a/2-a\right)}$$

$$Area=6\sqrt{(3a/2) \left(a/2\right)\left(3a/2\right)\left(3a/2\right)}$$

$$Area=\frac{3\sqrt{3}a^{2}}{2}$$

## Polygon

n = number of sides
s = length of a side
r = radius of the inscribed circle

$$Area=\frac{nsr}{2}$$

The number of triangles (when you draw all the diagonals from one vertex) in a polygon = (n - 2). We know each triangles interior angle is 1800. Hence

Exterior

 angle

Interior angle

2r

s

Sum of the interior angles = (n - 2) x 180°

The number of diagonals in a polygon = n (n -3)/2

# Shapes 3D

## Cone



$Surface Area=πrS+ πr^{2}$

$$Volume=\frac{πr^{2}h}{3}$$

## Pyramid

For symmetric square pyramid

$$Surface Area=Base Area+ \frac{Base perimeter X Slant height}{2}$$

$$Volume=\frac{Base Area X height}{3}$$