QuestionPack - Math Number & Counting

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9/23/2007

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PRIME NUMBERS

Prime numbers have only two factors, 1 and the number itself. List of prime numbers less than 500.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499

ROMAN NUMERALS

Values for different symbols used in Roman numerals are shown below.

Symbol	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

These symbols are combined to produce different values. When combining these symbols some rules have to be observed. At most 3 consecutive symbols of the same kind can be placed. The symbols can be repeated are I, X, C and M. By have two 'I' to form II represent a value of two. Like III represents 3. So is XX is 20, XXII is 22.

If a symbol with a smaller value appears before the symbol with higher value it is considered as subtraction. For example IV is 4, IX is 9, XC is 90, and XCII is 92.

Finally for large numbers, add a bar on top to indicate it is multiplied by 1000. $\overline{V}$ is 5000, $\overline{X}$ is 1000 and so on.

NUMBERS WITH DIFFERENT BASES

BASE N TO DECIMAL CONVERSION

What we normally use is called decimal number system which is also base 10. The value of a digit depends on the position where it occurs.

$$value\ of\ a\ digit = digit \times base^{position}; where$$

$$position\ is\ the\ zero\ based\ index\ from\ the\ right, where\ the\ digit\ occurs\ in\ a\ given\ number$$

$$base\ is\ the\ base\ of\ the\ number\ system. For\ decimal\ number\ system\ it\ is\ 10.$$

(e.g.)

$$2941_{10} = 2 \times 10^3 + 9 \times 10^2 + 4 \times 10^1 + 1 \times 10^0$$

$$1347_8 = 1 \times 8^3 + 3 \times 8^2 + 4 \times 8^1 + 7 \times 8^0$$

In a base n, number, each digit can be of 0 to n – 1. So in base 8, you cannot have a digit 8 or 9. In base 6 digit, individual digits can range from 0 to 5.

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Table showing various bases for the first 16 numbers

Base 10 (Decimal)	Base 2 (Binary)	Base 6	Base 8	Base 16 (Hexadecimal)
0	0	0	0	0
1	1	1	1	1
2	10	2	2	2
3	11	3	3	3
4	100	4	4	4
5	101	5	5	5
6	110	10	6	6
7	111	11	7	7
8	1000	12	10	8
9	1001	13	11	9
10	1010	14	12	A
11	1011	15	13	B
12	1100	20	14	C
13	1101	21	15	D
14	1110	22	16	E
15	1111	23	17	F
16	10000	24	20	10

DECIMAL TO BASE N CONVERSION

Keep dividing the decimal number by n, until no more. Append the reminders from last to first, the resulting number is base n.

(e.g.) Convert 692_{10} to base 6.

$$\frac{692}{6}\ Quotient = 115, Reminder = 2$$

$$\frac{115}{6}\ Quotient = 19, Reminder = 1$$

$$\frac{19}{6}\ Quotient = 3, Reminder = 1$$

Hence the Base 6 number is 3112_6. You can verify this by

$$3 \times 6^3 + 1 \times 6^2 + 1 \times 6^1 + 2 \times 6^0 = 692$$

SERIES

ARITHMETIC SERIES

In the Arithmetic series difference between any two successive members is a constant. Let us d is the difference between two consequently members, a is the starting number.

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-1)d)$

$$\boldsymbol{S_n = \frac{a_1 + a_n}{2} \times n}$$

Alternate way of writing the same equation $a_n = a_1 + (n-1)d$

$$\boldsymbol{S_n = \frac{2a_1 + (n-1)d}{2} \times n}$$

Because, $a_n = (n-1)d + a, \textit{where a is the starting number}$

Proof:

Add S_n twice by reversing the series as shown below.

$S_n \quad = a_1 \quad + (a_1 + d) \quad + (a_1 + 2d) \quad + \cdots + (a_1 + (n-1)d)$

$S_n \quad = (a_1 + (n-1)d) \quad + (a_1 + (n-2)d) \quad + (a_1 + (n-3)d) \quad + \cdots + \ a_1$

Adding the above equations

$2 \times S_n = (2a_1 + (n-1)d) + (2a_1 + (n-1)d) + (2a_1 + (n-1)d) + \cdots + \ (2a_1 + (n-1)d)$

$2 \times S_n = (2a_1 + (n-1)d) \times n$

$$S_n = \frac{(2a_1 + (n-1)d)}{2} \times n$$

SUM OF SQUARES

$\boldsymbol{1^2 + 2^2 + 3^2 \ldots + n^2 = n(n+1)(2n+1)/6}$

SUM OF CUBES

$\boldsymbol{1^3 + 2^3 + 3^3 \ldots + n^3 = n^2(n+1)^2/6}$

GEOMETRIC SERIES

In the geometric series the ratio between any two successive terms is a constant. And r > 0 and r < 1

$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$

$r \times S = r \times [a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}]$

Subtracting above equations

$S(1-r) = a - ar^n$

$$S = \frac{a(1-r^n)}{1-r}$$

For an infinite series, where $n \to \infty$, The equation simplifies to

$$S = \frac{a}{1-r}$$

COUNTING SQUARES AND TRIANGLES

SQUARE COUNTING

SQUARE

How many squares are there in 8 x 8 square?

Number of squares of size 8 x 8 = 1 = 1^2

Number of squares of size 7 x 7 = 4 = 2^2

Number of squares of size 6 x 6 = 9 = 3^2

Number of squares of size k x k = $(9\text{-}k)^2$

Total Number of squares = $1^2 + 2^2 + 3^2 + \ldots + 8^2$ = 8 (8+1)(2x8+1)/6 = 204

For a square of size of size n x n

Number of squares of size k x k = $(n+1-k)^2$

$$\textbf{\textit{Total Number of squares}} = \boldsymbol{n(n+1)(2n+1)/6}$$

RECTANGLE

Number of rectangles of size j x k in a rectangle of size m x n = (m+1-j)(n+1-k)

Total number of rectangles in m x n = number of ways 2 edges can be chosen out of m+1 X number of ways 2 edges can be chosen from n+1 edges

$$\binom{m+1}{2} \times \binom{n+1}{2} = \frac{(m+1)m}{2} \times \frac{(n+1)n}{2}$$

$$\frac{(m+1)m(n+1)n}{4}$$

TRIANGLE COUNTING

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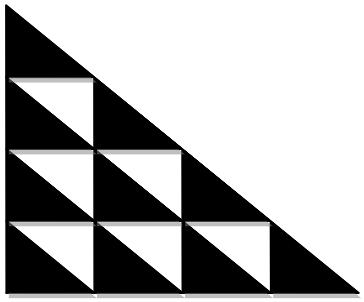
Counting the number of all possible triangles triangles given a large triangle of size n x n. The picture shows a triangle of size 4 x 4.



Number of up triangles (like black) of size 1 = 10, size 2=6, size 3=7, size 4=1

Number of down triangles (like white) of size 1=6, size 2=1

For up triangles, number of triangles of various sizes formed is given by

		Size of triangle formed					
		1	2	3	4	5	6
Outer triangle size (n)	1	1					
	2	2+1=3	1				
	3	3+2+1=6	2+1=3	1			
	4	4+3+2+1=10	3+2+1=6	2+1=3	1		
	5	5+4+3+2+1=15	4+3+2+1=10	3+2+1=6	2+1=3	1	
	6	6+5+4+3+2+1=21	5+4+3+2+1=15	4+3+2+1=10	3+2+1=6	2+1=3	1

For down triangles, number of triangles of various sizes formed is given by

		Size of triangle formed					
		1	2	3	4	5	6
Outer tr iangle size (n)	1	0					
	2	1					
	3	2+1=3					
	4	3+2+1=6	1				
	5	4+3+2+1=10	2+1=3				
	6	5+4+3+2+1=15	3+2+1=6	1			

Total number is obtained adding corresponding rows from above 2 tables. It can be represented using the following formula

$$\textit{Total Number of Triangles} = \begin{cases} \dfrac{n(n+2)(2n+1)}{8} & \textit{When } n \textit{ is even} \\ \dfrac{n(n+2)(2n+1)-1}{8} & \textit{When } n \textit{ is odd} \end{cases}$$