'Q'uestionPack Monthly Magazine

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MATHCOUNTS

MATHCOUNTS is one of the important national math competitions for the middle school students. Students enrolled in 6^{th}, 7^{th} and 8^{th} grades are eligible to participate in the competition. About 40,000 students from all 50 states and U.S. territories participate in the annually in the local competition. Consisting of individual and team rounds and an oral round, it is a fun contest testing the children in problem solving and critical thinking areas. MATHCOUNTS releases two volumes of school handbooks each year with practice problems to get ready for the competition.

There are four different levels of this competition – School, Chapter, State and National. School round is generally conducted by the school math club coach. This test could be the one the coach receives from the MATHCOUNTS or the one coach prepares himself. Since the school round consists of easy questions, coaches prepare a harder test, about chapter or state level, to help with fair selection process. Four top scoring students for a team and next six (used to be four until last year) students are selected as individuals representing the school.

Chapter round is conducted for the winners of the school round from all the schools in that chapter. A winning team of four and four top scoring individuals progress to the state round. In the state round, winners from all the chapters in your state compete and the top scoring four individuals are selected to represent their state in the National competition.

MATHCOUNTS is approximately a 3 hour competition. There are Sprint, Target, Team and Countdown rounds. Countdown is an optional round at Chapter and State levels whereas it is an official round at the national competition. The Sprint round has 30 questions to be completed in 40 minutes, Target has 8 questions, which are distributed 2 at a time and students have 6 minutes to complete each pair and 10 problems to be completed in 20 minutes. The top 25%, up to a maximum of 10 students proceed to the countdown round, an oral round where they compete one on one.

National round is going to be held in Washington DC from May 5^{th} to 8^{th}, 2011. Apart from the cash prizes, for the top individual winner, runner-up and team, the most lucrative award is the recognition by the president at the White House.

For further questions, visit their official website www.mathcounts.org

Mock Sprint Round

40 minutes, no calculator

(Answers on Page 6)

1. Gary buys water, soda, and lemonade in a ratio of 2:3:4 ounces. If he bought 27 ounces of liquid, how many ounces of water did he buy?
2. How many ways can you arrange 3 distinguishable books on a shelf?
3. Jan bought a TV that was on sale for 75% of the original price. If the original price was $360.00 then how much did Jan spend on the TV? Express your answer in dollars.
4. Find the area of a rhombus with diagonals of length 4cm and 12cm. Express your answer in cm^2
5. If 8 bogs are equal to 7 cogs, and 12 nogs are equal to 1 cog, then how many nogs are in 16 bogs.
6. If Robert was born in 1995 and Sandra was born in 2004, then on what year did Robert become 4 times as old as Sandra?
7. Lisa tossed a fair coin 5 times. What is the probability of tossing at least 3 heads? Express your answer in reduced simple fraction.
8. George's favorite number is between 10 and 60. If George's favorite number is divided by 5 the remainder is 4 and if it is likewise divided by 7 the remainder is 4. What is George's favorite number?
9. There are 40 white marbles and 20 black marbles in Jar A, and there are 30 black marbles and 45 white marbles in Jar B. If a jar is randomly chosen, what is the probability of picking a white marble on the first pick? Assume both jars have an equal probability of being chosen. Express your answer in reduced simple fraction.
10. Nancy was practicing to run a marathon. On the first week she ran 1 mile per day. On the second week she ran 2 miles per day. This pattern continued until the 10^{th} week during which she ran 10 miles per day. What is the total number of miles that Nancy ran?
11. In farmer Bob's yard, he has 40 feet of fencing which he uses to make a rectangular pen with integral side lengths for his cows. Find the positive difference between the greatest and least possible areas of the pen. Express your answer in terms of $ft.^2$
12. Claudia, Richard, and Andrew start reading a book at the same time. Claudia reads 4 pages per minute. Richard reads 2 pages per minute, and Andrew reads 5 pages per minute. How many seconds elapse before they finish reading a page at the same time?
13. Car A leaves City B at 9:00 a.m. and heads toward City A at a constant speed of 35 mph. One hour later Car B leaves City A and heads toward City B at a constant speed of 45 mph. If the cities are 355 miles apart, at what time will they meet? Include a.m. or p.m. in your answer.
14. John takes 15 minutes to bike from his home to school, and he takes 1 hour to come back home along the same route by walk. If the distance between John's house and school is 4 miles, what is his average speed in miles per hour? Express your answer in decimal form.
15. If 4 chickens lay 24 eggs in 3 days then how many days would it take for 11 chickens to lay 110 eggs?

16. The Sounders have currently won 60% of their baseball games. Later they win 6 more games in a row and their winning percentage becomes 70%. How many games have they played so far?
17. $a \emptyset b = \frac{a+b}{a^2-ab}$ and $a \aleph b = b^2 - a^2$ then find $2\aleph(2\emptyset 6)$
18. How many ounces of pure chocolate must be added to 40 ounces of a 70% chocolate milk drink to make it a 75% chocolate milk drink?
19. How many integer multiples of 7 are between 1700 and 2700?
20. What is the positive difference of the sum of the first 25 even integers and the sum of the first 35 odd integers?
21. Which of the following is the greatest: 2^{30}, 3^{20}, or 4^{15}?
22. Richard spends a total of $185 on hats and shoes. If hats cost $19 and pairs of shoes cost $21 then how many pairs of shoes did Richard buy?
23. Fred loves animals and has many creatures as pets. He owns jellyfish, squid, and cows. A jellyfish has 1 head and 8 tentacles, a squid has no head and 12 tentacles, and a cow has 4 legs and 1 head. If there are 11 heads, 96 tentacles, and 20 legs (not including Fred), then how many squids are there?
24. Bob's alarm clock has been behaving strangely starting from 12:30 a.m. Every 1 hour it gains 12 minutes at a constant rate. If the alarm clock goes off at 6:30 a.m., how many minutes early did it ring? Express your answer as a decimal.
25. An irregular tetrahedron with height 1 cm and a base area of 3 cm^2 is cut from each vertex of a cube with side length 10 cm. What is the volume of the new 14-face object? Express your answer in terms of cm^3.
26. An equilateral triangle is inscribed in a unit circle. Find the area outside the triangle, and inside the circle. Express the area in terms of pi in the simplest radical form.
27. A ping pong ball is dropped on a marble floor from a height of 2 meters. After every bounce, the ball bounces back 80% of the previous height. What is the total distance the ball travels before it comes to a rest? Express your answer in centimeters.
28. A convex hexagon is inscribed in a circle and another hexagon is circumscribed about the same circle. What is the ratio of the area of the smaller hexagon to the area of the larger hexagon? Express your answer as a reduced common fraction.
29. Mr. Ant is on a vertex of a rectangular prism with dimensions 4" x 6" x 9" and wants to get to the vertex farthest from him. He wants to get there in the shortest possible path. If he can walk along the face and edges, what is the least distance he must travel to get to the honey? (Express your answer in simplest radical form in inches.)
30. What is the minimum number of weights to measure any integral value from 1 to 12 pounds inclusive using a common balance? You may add and subtract weights.

Stars and Bars

This technique is used to find the number of ways to partition *n* indistinguishable objects into *k* groups. Take for example, 5 objects, which need to be partitioned into 3 groups. To divide the given objects into 3 groups, you need 2 bars (dividers). Two possible partitions are given below:

1. ** | * | ** ← Group 1 has 2 objects, Group 2 has 1 object, Group 3 has 2 objects.
2. ** || *** ← Group 1 has 2 objects, Group 2 has zero objects, Group 3 has 3 objects.

Essentially, the problem boils down to the number of ways to place *x* bars into *y* positions. The number of positions the bar can be placed equals the sum of the number of bars and the number of stars. The example mentioned before, with 5 objects and 2 bars, is given by $\binom{7}{2} = \frac{7!}{5!2!} = \frac{7*6}{2*1} = 21$. For the generic case of dividing n objects into k groups, we need (k-1) bars. So, the total number of locations that we can place $(k-1)$ bars into is $n + (k-1)$. The number of such possibilities equals $\binom{n+k-1}{k-1}$.

In case you didn't already know what $\binom{x}{y}$ stands for, $\binom{x}{y} = \frac{x!}{y!(x-y)!}$ and is pronounced "x choose y." (i.e.) it represents number of ways to choose y objects given x objects, where order is not important.

Stars and Bars - Sample problems

Example 1: How many ways you can distribute 5 coins between 4 people?

Solution: Converting this to stars and bars, we need 5 stars and 3 bars. For example,

** | | * | **

The above representation partitions two coins for Person #1, zero coins for Person #2, one coin for Person #3, and two coins for Person #4. The number of ways to arrange the 3 bars is given by $\binom{8}{3} = \frac{8x7x6}{1x2x3} = 56$. Therefore, the total number of ways we can distribute 5 coins between 4 people is **56**.

Example 2: How many non-negative integral solution sets for a, b and c can satisfy the equation a+b+c=100?

Solution: This problem can be converted into the number of ways we can partition 100 ones ('1') into three sets. To form three sets, you need two bars. Adding 2 bars and 100 ones you have a total of 102 places. The number of ways these two dividers can be placed in the 102 places is given by $\binom{102}{2} = \frac{102x101}{2} = \mathbf{5151}$.

Example 3: How many integral solutions for a, b and c satisfy the equations $a + b + c = 100$ and a, b, c ≥ 5?

Solution: We approach the problem similar to the previous problem, the first condition is the same as the equation in Example #2. To satisfy the second condition, assign the 5 ones ('1') to each of a, b, c. Now the problem becomes how many ways to allocate the remaining 85 into 3 groups, which is given by $\binom{87}{2} = \frac{87x86}{2} = \mathbf{3741}$.

Example 4: Jessie went to buy flowers to make some bouquets. She can choose 4 different types of flowers. She plans to have 6 flowers in each bouquet. How many different combinations of bouquets she can make?

Solution: 6 flowers can be represented as 6 stars. We need 3 bars to distinguish 4 types of flowers. For example, * | | ***** | denotes Jessie is picking one flower of Type 1, zero flowers of Type 2, five flowers of Type 3 and zero flowers of Type 4. The number of ways the bars can be arranged is given by $\binom{9}{3} = \frac{9x8x7}{1x2x3} = \mathbf{84}$.

Stars and Bars - Exercises

1. Cara goes to a store and wants to buy flowers for her mother's birthday. There are 3 different types of flowers, tulips, roses, and lilies. If she wants exactly 11 flowers in all, how many different combinations can Cara buy?

2. For John's school supply list, he needs 8 mechanical pencils. He goes to a store and finds 3 different types of mechanical pencils. How many different combinations of pencils can John buy?

3. Mary loves seaweed and she goes to a store to buy some. She wants 5 packets of seaweed, and there are 4 different types. How many different combinations of seaweed can Mary buy?

4. Nancy loves taking pictures and she is going to 4 different sight-seeing places. Unfortunately there the memory in her camera can only hold 6 more pictures. If she uses up all the 6 pictures in the 4 sight-seeing places, how many different combinations are there for her to take pictures?

Answer Key (Mock Sprint)

1. 6 ounces
2. 6
3. $270
4. 24 cm^2
5. 168 nogs
6. 2007
7. $\frac{1}{2}$
8. 39
9. $\frac{19}{30}$
10. 385 miles
11. 81 $ft.^2$
12. 60 seconds
13. 2:00 p.m.
14. 6.4 mph
15. 5 days
16. 24 games
17. -3
18. 8 ounces
19. 143
20. 575
21. 3^{20}
22. 7 shoes
23. 4 squids
24. 60 minutes
25. 992 cm^3
26. $\pi - \frac{3\sqrt{3}}{4}$
27. 1000 centimeters
28. $\frac{3}{4}$
29. $\sqrt{181}$ inches
30. 3 weights (actual weights are 1, 3 and 9)

Stars and Bars – Answers for Exercises

1. 78
2. 45
3. 56
4. 84

Up Coming Math Competitions…

Math Competition	Date
American Mathematics Contest (AMC) 10A/12A	Feb 8th , 2011
Mathcounts – Chapter	Feb 1-28, 2011*
American Mathematics Contest (AMC) 10B/12B	Feb 23rd, 2011
Mathcounts – State	Mar 1-27, 2011*
American Invitational Mathematics Examination (AIME)	Mar 17th or 30th , 2011
Purple Comet Math Meet	Apr 4th - 10th, 2011**
Mathcounts – National	May 5-8, 2011
American Regions Math League (ARML)	June 3rd and 4th, 2011

*Test will be conducted on a Saturday in the given time frame. Please contact your local MATHCOUNTS coordinator for exact date, time and location.

** Test can be taken any time in the given time frame.